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H∞ Controller and Its Application in Electric Power System: A Review

Rudy Gianto
Department of Electrical Engineering, Tanjungpura University, Indonesia

Abstract: *H∞ controller is one of the techniques of robust control. H∞ control technique is aimed to obtain a robust control system, i.e. the control system is insensitive to the differences between the actual system and the model of the system which was used to design the controller. This control technique is very popular, and has been implemented in many important engineering applications. One important application of H∞ controller is in the field of electrical power engineering, where the H∞ controller concept has been employed for the purpose of damping of power oscillation. This paper provides an overview on the H∞ controller and its application in improving dynamic performance of an electrical power system.*
Keywords: *H∞ controller, robust control, dynamic performance, power system, oscillation damping*

I. Introduction

H∞ control theory was originally formulated by Zames in the early 1980s, and is aimed to obtain a robust control system [1]. A control system is robust if it is insensitive (i.e. remains stable and achieves certain performance criteria) to the differences between the actual system and the model of the system which was used to design the controller. These differences are referred to as *model uncertainty* [1-3]. Typical sources of the difference include unmodelled (usually high-frequency) dynamics, neglected nonlinearities in modeling, effects of deliberate reduced-order models and changes in system operating conditions [1-3]. In other words, the goal of the H∞ control theory is to obtain satisfactory performance specifications even for the "worst-case" of uncertainty.

The H∞ controller is developed to overcome the shortcomings of the non-robust controllers. The non-robust (fixed-parameter) controller is, in general, based on one particular system operating condition. The key disadvantage of this controller is that the possibility of the controller performance deterioration under other operating conditions. Furthermore, it is not possible to achieve maximum performance for each and every operating condition when the controller parameters are fixed. This paper provides an overview on the H∞ controller and its application in improving dynamic performance of an electrical power system [1-22].

In order to be systematic, this paper is organized as the following. First, the overview of the H∞ control theory will be discussed to introduce the terminologies used in the H∞ design framework and explain its principle. Then, the summary will be given of the published reports of the H∞ approaches for power system damping control which have been investigated over the last decade.

II. H∞ Control Theory

2.1. H∞ Norm

The H∞ norm has been extensively used in H∞ control problem formulation because it is the convenient way for representing the model uncertainty [1]. It is to be noted that, in H∞ design framework, the uncertainty can be modeled as perturbations to the nominal model. In H∞ controller design, the H∞ norm is minimized in order to obtain the robust design for the controller. It will be shown that minimizing this H∞ norm corresponds to minimizing the peak of the largest singular value ("worst direction, worst frequency"), and therefore, it can be used as a measure of the worst possible performance of the control system [1].

The H∞ norm of a system is the peak value of the transfer function magnitude over the whole frequency range. In a multi-input-multi-output (MIMO) system, the H∞ norm is the peak of the largest singular value and can be expressed as [1]:

$$\|\boldsymbol{G}(s)\|_\infty = \max_\omega \bar{\sigma}(\boldsymbol{G}(j\omega)) \tag{1}$$

Since the singular value provides maximum gain in the principal direction, H∞ norm can be seen as the magnitude of the transfer function in the worst direction over the entire frequency range [1].
The maximum singular value $\bar{\sigma}$ of transfer matrix G is determined by [1]:

$$\bar{\sigma}(\boldsymbol{G}) = \frac{\|\boldsymbol{G}\boldsymbol{v}_1\|_2}{\|\boldsymbol{v}_1\|_2} \tag{2}$$

where $\boldsymbol{v}_1$ is the vector of the first column elements of unitary matrix $\boldsymbol{V}$. The unitary matrix $\boldsymbol{V}$ can be found by using the singular value decomposition of $\boldsymbol{G}$, i.e. $\boldsymbol{G} = \boldsymbol{U\Sigma V}^H$ (note that the superscript $\boldsymbol{H}$ represents the matrix complex conjugate). In (2), $\|\cdot\|_2$ is a vector 2-norm and defined by [1, 3]:

$$\|\boldsymbol{x}\|_2 = \sqrt{|x_1|^2 + |x_2|^2 + \cdots} \tag{3}$$

where $|\boldsymbol{x}_i|$ is the magnitude of the ith element of vector $\boldsymbol{x}$.

2.2. Controller Design

This section discusses the *transfer function shaping approach* for controller design. In this approach, the designer specifies the "magnitude" of some transfer function(s) as a function of frequency, and then finds a controller which gives the desired shape(s) [1]. The transfer function shaping approach can be subdivided into two approaches as follows:

(i) *Loop-shaping approach*. This is the classical approach in which the magnitude of the open-loop transfer function is shaped. However, classical loop-shaping is difficult to apply for complicated systems, and therefore, the Glover-McFarlane H_∞ loop-shaping design is preferred instead.

(ii) *Closed-loop transfer function shaping approach*. In this approach, the closed-loop transfer functions such as S, T and KS are to be shaped in the design. Optimization is usually used in the approach, resulting in various H_∞ control problems such as mixed-sensitivity. The following is the explanation of the S, T and KS transfer functions.

Consider the standard feedback control system shown in Fig.1 [1, 2]. In Fig.1, G is the plant model, K is the controller model to be designed, r is the reference inputs (commands, set-points), d is the disturbances, n is the measurement noise, y is the plant outputs (these signals include the variables to be controlled), y_m is the measured y, u is the controller output signals (manipulated plant inputs), and v is the controller inputs (i.e. the difference between the reference inputs and measured plant outputs).

For the control system of Fig.1, it can be shown that the following relationships hold [1, 2]:

$$\boldsymbol{y} = \boldsymbol{Tr} + \boldsymbol{Sd} - \boldsymbol{Tn} \tag{4}$$

$$\boldsymbol{u} = \boldsymbol{KSr} - \boldsymbol{KSd} - \boldsymbol{KSn} \tag{5}$$

$$\boldsymbol{e} = \boldsymbol{y} - \boldsymbol{r} = -\boldsymbol{Sr} + \boldsymbol{Sd} - \boldsymbol{Tn} \tag{6}$$

where $\boldsymbol{S} = (\boldsymbol{I} + \boldsymbol{GK})^{-1}$ is the *sensitivity function*, and $\boldsymbol{T} = (\boldsymbol{I} + \boldsymbol{GK})^{-1}\boldsymbol{GK}$ is the *complementary sensitivity function*. It can be seen that S is the closed-loop transfer function from the disturbances to the outputs, while T is the closed-loop transfer function from the reference signals to the outputs.

The objective of the robust control design is to find a controller such that the closed-loop system is robust. As mentioned in the previous discussion, in order to achieve this, the H_∞ norm of the transfer matrix should be minimized. Similarly, for the control system shown in Fig.1, in order to obtain the best performance specifications such as disturbance rejection or noise attenuation for any r, d or n, the H_∞ norm of the corresponding transfer matrices should also be minimized.

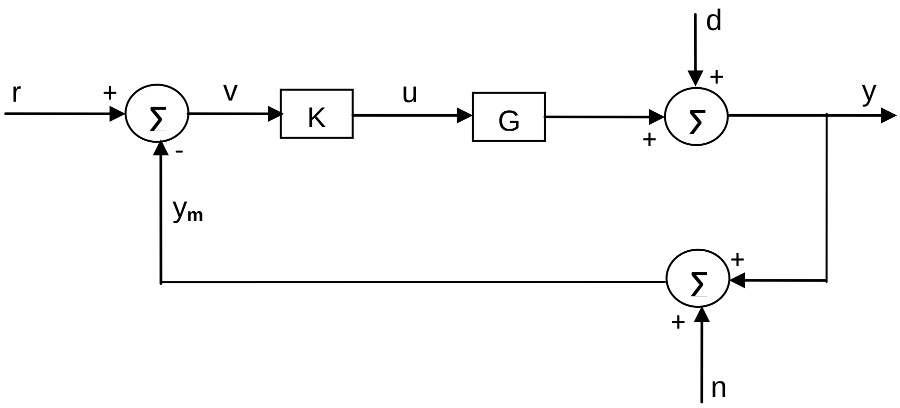

Figure 1 Standard feedback control system

Therefore, the controller design problem can be formulated as follows: over the set of all stabilizing controllers K's (i.e. those K's make the closed-loop system internally stable), find the optimal one that minimizes [1, 2]:

- $\|S\|_\infty$; for good disturbance rejection or tracking
- $\|T\|_\infty$; for good noise attenuation, and
- $\|KS\|_\infty$; for control energy reduction

2.3. Standard H∞ Control Problem

Fig.2 shows a general control configuration where any particular control problem can be manipulated into this configuration [1-3]. The standard control system in Fig.1 can be transformed into an equivalent form of the general structure in Fig.2 which is more convenient to formulate the H_∞ control problem. The system of Fig.2 is described by:

$$\begin{bmatrix} z \\ v \end{bmatrix} = P \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{7}$$

$$u = Kv \tag{8}$$

where P is the generalized plant model (this will include the plant model G and the interconnection structure between the plant and the controller), w is the exogenous inputs (commands, disturbances and noise), z is the exogenous outputs ("error" signals to be minimized to meet the control objectives, i.e. y – r).

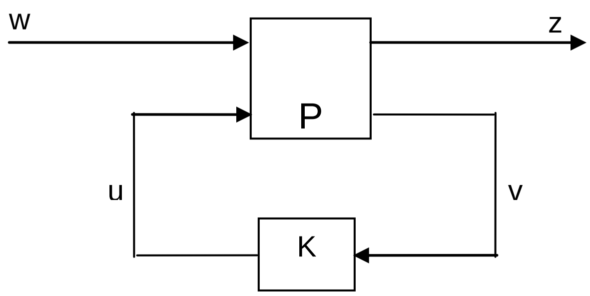

Figure 2 General control configuration

In state-space approaches to H_∞ control, it is common to introduce the realization of the generalized plant P in the form of [4]:

$$P = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} (sI - A)^{-1} \begin{bmatrix} B_1 & B_2 \end{bmatrix} \tag{9}$$

This realisation corresponds to the state-space equations:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ v &= C_2 x + D_{21} w + D_{22} u \end{aligned} \tag{10}$$

Assume that the realization of the controller to be determined in Fig.2 is: $K = D_K + C_K (sI - A_K)^{-1} B_K$, and the corresponding state-space equations is of the form:

$$\begin{aligned} \dot{x}_K &= A_K x_K + B_K v \\ u &= C_K x_K + D_K v \end{aligned} \tag{11}$$

With the generalized plant defined as (10) and the controller model defined as (11), it can be shown that the realization of the closed-loop system shown in Fig.2 in state-space form is given by [4, 5]:

$$\begin{aligned} \dot{x}_{cl} &= A_{cl}\, x_{cl} + B_{cl} w \\ z &= C_{cl}\, x_{cl} + D_{cl} w \end{aligned} \tag{12}$$

where:

$$\begin{aligned} A_{cl} &= \begin{bmatrix} A + B_2 D_K C_2 & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix} \\ B_{cl} &= \begin{bmatrix} B_1 + B_2 D_K D_{21} \\ B_K D_{21} \end{bmatrix} \\ C_{cl} &= \begin{bmatrix} C_1 + D_{12} D_K C_2 & D_{12} C_K \end{bmatrix} \\ D_{cl} &= D_{11} + D_{12} D_K D_{21} \end{aligned} \tag{13}$$

The results in (13) have been obtained by assuming D_{22} in (10) equal to 0. This assumption will incur no loss of generality and has been made only to simplify the calculations [4]. It can also be shown that the closed-loop transfer function from w to z for the system configuration in Fig.2 is given by [2-4]:

$$TF_{zw} = D_{cl} + C_{cl}(sI - A_{cl})^{-1} B_{cl} \tag{14}$$

The standard H_∞ optimal control problem is to find all stabilizing controllers which minimize the H_∞ norm of the closed-loop transfer function:

$$\left\| TF_{zw} \right\|_\infty = \max_{\omega} \bar{\sigma}\left(TF_{zw}(j\omega)\right) \tag{15}$$

In practice, it is usually not necessary to obtain an optimal controller for the H_∞ problem, and it is often simpler to design a sub-optimal one. Therefore, the H_∞ sub-optimal control problem consists of finding all stabilizing controllers such that [2-4]:

$$\left\| TF_{zw} \right\|_\infty < \gamma \tag{16}$$

where γ is greater than the minimum value of $\left\| TF_{zw} \right\|_\infty$ over all stabilizing controllers.

The standard H_∞ optimal control problem (16) can be solved by: (i) analytical approach using a positive semi-definite solution to the algebraic Ricatti equations (AREs), or (ii) numerically optimize certain performance index such that the algebraic Ricatti inequalities (ARIs) are satisfied. Although ARIs are nonlinear, they can be converted into linear matrix inequalities (LMIs) by using linearization techniques [6, 7].

The numerical approach using LMIs has a distinct advantage as additional constraints (such as minimum damping ratio) can be included in the design in a straight forward manner [6]. In order to ensure a minimum damping ratio, a method known as pole-placement is used in the design. In the method, the poles of the closed-loop system are placed within a certain region in the complex plane. LMI-based solution to the H_∞ control problem is described in the following section.

2.4. LMI-Based H□ Design

By using the *Bounded Real Lemma* and the *Schur's formula*, it can be concluded that the H_∞ constraint (16) is equivalent to the existence of a solution of a symmetric matrix $X_{cl} > 0$ to the following matrix inequality [4, 7]:

$$\begin{bmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & B_{cl} & X_{cl} C_{cl}^T \\ B_{cl}^T & -\gamma I & D_{cl}^T \\ C_{cl} X_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \tag{17}$$

In the matrix inequality (17), $\boldsymbol{A}_{cl}$, $\boldsymbol{B}_{cl}$, $\boldsymbol{C}_{cl}$ and $\boldsymbol{D}_{cl}$ are functions of the controller variables $\boldsymbol{A}_K$, $\boldsymbol{B}_K$, $\boldsymbol{C}_K$ and $\boldsymbol{D}_K$, and the controller variables are functions of $\boldsymbol{X}_{cl}$. This makes the products of the terms involving $\boldsymbol{X}_{cl}$ in (17) nonlinear. The following techniques are used to change the controller variables and convert the problem into a linear one.

Let n be the number of the plant states (size of $\boldsymbol{A}$) and k be the order of the controller (with $k \le n$), and also let $\boldsymbol{X}_{cl}$ (of dimension $(n+k)\times(n+k)$) and its inversion ($\boldsymbol{X}_{cl}^{-1}$) be partitioned as:

$$\boldsymbol{X}_{cl} = \begin{bmatrix} \boldsymbol{R}_H & \boldsymbol{M}_H \\ \boldsymbol{M}_H^T & \boldsymbol{U}_H \end{bmatrix}; \boldsymbol{X}_{cl}^{-1} = \begin{bmatrix} \boldsymbol{S}_H & \boldsymbol{N}_H \\ \boldsymbol{N}_H^T & \boldsymbol{V}_H \end{bmatrix} \tag{18}$$

where $\boldsymbol{S}_H$ and $\boldsymbol{R}_H$ are of dimension $n \times n$ and symmetric. It can be shown that $\boldsymbol{X}_{cl}$ will satisfy the identity $\boldsymbol{X}_{cl}\boldsymbol{\Pi}_2 = \boldsymbol{\Pi}_1$ for [5, 7]:

$$\boldsymbol{\Pi}_1 = \begin{bmatrix} \boldsymbol{R}_H & \boldsymbol{I} \\ \boldsymbol{M}_H^T & \boldsymbol{0} \end{bmatrix}; \boldsymbol{\Pi}_2 = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{S}_H \\ \boldsymbol{0} & \boldsymbol{N}_H^T \end{bmatrix} \tag{19}$$

Also, let the new controller variables be defined as:

$$\begin{aligned} \hat{\boldsymbol{A}} &= \boldsymbol{N}_H \boldsymbol{A}_K \boldsymbol{M}_H^T + \boldsymbol{N}_H \boldsymbol{B}_K \boldsymbol{C}_2 \boldsymbol{R}_H + \boldsymbol{S}_H (\boldsymbol{A} + \boldsymbol{B}_2 \boldsymbol{D}_K \boldsymbol{C}_2) \boldsymbol{R}_H \\ \hat{\boldsymbol{B}} &= \boldsymbol{N}_H \boldsymbol{B}_K + \boldsymbol{S}_H \boldsymbol{B}_2 \boldsymbol{D}_K \\ \hat{\boldsymbol{C}} &= \boldsymbol{C}_K \boldsymbol{M}_H^T + \boldsymbol{D}_K \boldsymbol{C}_2 \boldsymbol{R}_H \\ \hat{\boldsymbol{D}} &= \boldsymbol{D}_K \end{aligned} \tag{20}$$

By examining the identity $\boldsymbol{X}_{cl}\boldsymbol{X}_{cl}^{-1} = \boldsymbol{I}$ or $\boldsymbol{X}_{cl}\boldsymbol{\Pi}_2 = \boldsymbol{\Pi}_1$, it can be shown that:

$$\boldsymbol{M}_H \boldsymbol{N}_H^T = \boldsymbol{I} - \boldsymbol{R}_H \boldsymbol{S}_H \tag{21}$$

Pre- and post-multiplying the inequality $\boldsymbol{X}_{cl} > \boldsymbol{0}$ by $\boldsymbol{\Pi}_2^T$ *and* $\boldsymbol{\Pi}_2$ respectively leads to the following LMI problem [5, 7], the solution of which is used for forming $\boldsymbol{X}_{cl}$.

$$\begin{bmatrix} \boldsymbol{R}_H & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{S}_H \end{bmatrix} > \boldsymbol{0} \tag{22}$$

Similarly, pre- and post-multiplying the inequality (17) by $diag(\boldsymbol{\Pi}_2^T, \boldsymbol{I}, \boldsymbol{I})$ and $diag(\boldsymbol{\Pi}_2, \boldsymbol{I}, \boldsymbol{I})$ respectively, and carrying out appropriate change of variables according to (20), the following LMI is obtained [5, 7] for determining the controller variables:

$$\begin{bmatrix} \boldsymbol{\Psi}_{11} & \boldsymbol{\Psi}_{21}^T \\ \boldsymbol{\Psi}_{21} & \boldsymbol{\Psi}_{22} \end{bmatrix} < \boldsymbol{0} \tag{23}$$

where:

$$\begin{aligned} \boldsymbol{\Psi}_{11} &= \begin{bmatrix} \boldsymbol{A}\boldsymbol{R}_H + \boldsymbol{R}_H \boldsymbol{A}^T + \boldsymbol{B}_2 \hat{\boldsymbol{C}} + \hat{\boldsymbol{C}}^T \boldsymbol{B}_2^T & \boldsymbol{B}_1 + \boldsymbol{B}_2 \hat{\boldsymbol{D}} \boldsymbol{D}_{21} \\ (\boldsymbol{B}_1 + \boldsymbol{B}_2 \hat{\boldsymbol{D}} \boldsymbol{D}_{21})^T & -\gamma \boldsymbol{I} \end{bmatrix} \\ \boldsymbol{\Psi}_{21} &= \begin{bmatrix} \hat{\boldsymbol{A}} + (\boldsymbol{A} + \boldsymbol{B}_2 \hat{\boldsymbol{D}} \boldsymbol{C}_2)^T & \boldsymbol{S}_H \boldsymbol{B}_1 + \hat{\boldsymbol{B}} \boldsymbol{D}_{21} \\ \boldsymbol{C}_1 \boldsymbol{R}_H + \boldsymbol{D}_{12} \hat{\boldsymbol{C}} & \boldsymbol{D}_{11} + \boldsymbol{D}_{12} \hat{\boldsymbol{D}} \boldsymbol{D}_{21} \end{bmatrix} \\ \boldsymbol{\Psi}_{22} &= \begin{bmatrix} \boldsymbol{A}^T \boldsymbol{S}_H + \boldsymbol{S}_H \boldsymbol{A} + \hat{\boldsymbol{B}} \boldsymbol{C}_2 + \boldsymbol{C}_2^T \hat{\boldsymbol{B}}^T & (\boldsymbol{C}_1 + \boldsymbol{D}_{12} \hat{\boldsymbol{D}} \boldsymbol{C}_2)^T \\ \boldsymbol{C}_1 + \boldsymbol{D}_{12} \hat{\boldsymbol{D}} \boldsymbol{C}_2 & -\gamma \boldsymbol{I} \end{bmatrix} \end{aligned} \tag{24}$$

Therefore, the LMI-based solution to the H_∞ control problem consists of the following steps:

- Solve the LMIs (22) and (23) for $\boldsymbol{R}_H, \boldsymbol{S}_H, \hat{\boldsymbol{A}}, \hat{\boldsymbol{B}}, \hat{\boldsymbol{C}}$ and $\hat{\boldsymbol{D}}$
- Compute $\boldsymbol{M}_H$ and $\boldsymbol{N}_H^T$ by using a full-rank factorisation of $\boldsymbol{M}_H \boldsymbol{N}_H^T = \boldsymbol{I} - \boldsymbol{R}_H \boldsymbol{S}_H$
- Based on (20), determine the controller variables as follows:

$$\boldsymbol{D_K} = \hat{\boldsymbol{D}}$$

$$\boldsymbol{C_K} = \left(\hat{\boldsymbol{C}} - \boldsymbol{D_K C R}_H\right)\left(\boldsymbol{M}_H^T\right)^{-1}$$

$$\boldsymbol{B_K} = \boldsymbol{N}^{-1}\left(\hat{\boldsymbol{B}} - \boldsymbol{S}_H \boldsymbol{B D_K}\right)$$

$$\boldsymbol{A_K} = \boldsymbol{N}^{-1}\left(\hat{\boldsymbol{A}} - \boldsymbol{N B_K C R}_H - \boldsymbol{S B C_K M}_H^T - \boldsymbol{S}_H\left(\boldsymbol{A} + \boldsymbol{B D_K C}\right)\boldsymbol{R}_H\right)\left(\boldsymbol{M}_H^T\right)^{-1}$$

- Determine the controller transfer function using $\boldsymbol{K} = \boldsymbol{D_K} + \boldsymbol{C_K}(s\boldsymbol{I} - \boldsymbol{A_K})^{-1}\boldsymbol{B_K}$

III. Application Of H∞ Control In Power System

H_∞ approaches for power system damping control have been investigated over the last few decades [8-22]. The results of the investigation have also been reported in many literatures and can be summarized as follows:

- In [8], the methodology for the design of robust damping controllers for PSSs has been discussed. The design procedure was based on a formulation of the output feedback control problem, which is suited for damping controller design. With this formulation, the design problem can be expressed directly in the form of LMIs. Also, the inclusion of a regional pole placement criterion, as the design objective, allows the specification of a minimum damping factor for all modes of the controlled system. It has been shown in [8] that the controller is able to provide adequate damping for the oscillation modes of interest.
- In [9, 10], the design of an H_∞ controller for FACTS device for enhancing the electromechanical mode damping has been presented. The H_∞-based design procedure has been developed in an attempt to obtain a robust damping controller for a thyristor controlled series compensator (TCSC). In the procedure, two Riccati equations were used and solved in order to obtain the solution for the H_∞ optimization problem.
- The design process and a method to formulate the H_∞ optimal PSS design problem in terms of a general H_∞ control design framework have been discussed in [11]. The H_∞ design problem has been solved by using two algebraic Riccati equations. The H_∞-based PSS was tested by simulation on a single-machine infinite bus (SMIB) system. Results of the testing show that the proposed H_∞ PSS satisfies the design specifications.
- Design of a robust supplementary controller for a static VAr compensator (SVC) to improve the damping of a two-machine power system has been proposed in [12]. In the paper, the formulation of the damping control problem has been based on the H_∞ optimization. The solution to the design problem was obtained by solving the standard mixed-sensitivity control problem.
- In [13, 14], a Glover-McFarlane H_∞ loop-shaping approach has been used to design a robust control for a FACTS device and PSS respectively. In [13], the H_∞ loop-shaping was used to design a robust control for static compensator (STATCOM), series power flow controller (SPFC), voltage source converter (VSC)-based static phase shifter (SPS) and unified power flow controller (UPFC). The simulation has been carried out in [13] to show the effectiveness of the proposed controllers in improving the system damping. In [14], it has been shown that the H_∞ PSS can guarantee the stability of a set of perturbed plants with respect to the nominal system and exhibit a good oscillation damping ability.
- In [15], a method for designing low-order controllers for damping power swings has been proposed. The method was based on an H_∞ design formulation and uses LMI solver to obtain controller parameters. In particular, the proposed method has been used for design of a PSS for a SMIB system and a decentralized control for a TCSC and an SVC in a three-area system. Although the proposed method might not guarantee to provide global convergence, the convergence to a good damping controller design can be achieved [15].
- In [16], a robust design and tuning of a PSS in a SMIB system has been presented. In the method, maintaining stability and performance over a range of uncertain plant parameters (due to variations in generation and load patterns) was handled by imposing an upper bound on the H_∞ norm of the closed-loop transfer function. A pole region constraint was also included in the design. The solution to these design problems has been obtained by solving a standard LMI formulation.
- An H_∞ mixed-sensitivity design of a damping device employing a UPFC has been presented in [17]. The problem is posed in the LMI framework. The controller design was aimed at providing adequate damping to interarea oscillations over a range of operating conditions. The results obtained in a two-area four-machine test system have shown to be satisfactory both in the frequency domain and through nonlinear simulations.

- In [18, 19], a design procedure for robust damping controller of superconducting magnetic energy storage (SMES) device has been presented. The mixed-sensitivity H_∞ design based on the LMI formulation was used in the power system damping control design. Furthermore, a regional pole placement objective was also addressed in the design process [18].
- A H_∞ damping control design based on the mixed-sensitivity formulation in an LMI framework has been carried out in [20, 21]. In [20], a power system containing a controllable series capacitor (CSC), a static VAr compensator and a controllable phase shifter (CPS) was considered. It has been shown in [20] that the H_∞ controllers designed for these devices can improve the damping of interarea oscillation and also robust in the face of operating condition changes such as: varying power-flow patterns, load characteristics and tie-line strengths. In [21], a multiple-input single-output (MISO) H_∞ controller has been designed for a TCSC to improve the damping of the critical interarea modes. Also, in [21], the stabilizing signals are obtained from remote locations based on observability of the critical modes.
- The application of loop-shaping technique in H_∞ damping control design has been proposed in [22]. In [22], the problem of robust stabilization of a normalized coprime factor plant description was converted into a generalized H_∞ problem. The problem was solved using LMIs with additional pole-placement constraints. In addition to robust stabilization of the shaped plant, a minimum damping ratio can thus be ensured for the critical modes. The proposed method has been used to design a supplementary damping controller for a TCSC.

IV. Conclusion

This paper has presented and discussed the robust control technique, namely H_∞ controller. This control technique is very popular and has been implemented in many important engineering applications. One important application of H_∞ controller is in the field of electrical power engineering, where the H_∞ controller concept has been employed for the purpose of damping of power oscillation. The H_∞ controller is developed to overcome the shortcomings of the non-robust controllers, and is aimed to obtain satisfactory performance specifications even for the worst-case of system uncertainty. The application of the robust controller in enhancing power system dynamic performance has also been presented in the paper.

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